ELETTROTECNICA

MUSOLINO (DESTEC, sotto dip. informatica)

- M. RAUGI: "Lezioni di elettrotecnica" Ed. PBS
- A. LANGO: "Analisi delle reti elettriche" CEU
- F. BERTONCINI "Esercizi di elettrotecnica" Ed. PBS

4 CONSEGNE IN UN ANNO

- musolino@dsea.unipi.it orchestrapisa@dsea.unipi.it

- RICEVIMENTO 15.30 - 19.00 GIOVEDÌ

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MAXWELL $\nabla \cdot \vec{B} = 0 \longrightarrow \oint \vec{B} \cdot \vec{n}\, dS = 0$ (TEOREMA DELLE DIVERGENZE)

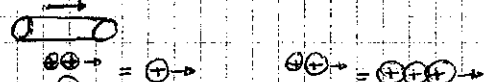
$\nabla \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow \oint \vec{B}_{fm} \cdot d\vec{s} = I$

$\nabla \times \vec{H} = I$ (con densità) $\longrightarrow \oint \vec{E} \cdot d\vec{s} = -\partial\Phi/\partial t$

$\oint \vec{J} \cdot d\vec{S} = -\frac{\partial Q}{\partial t}$ EQUAZIONE DI CONSERVAZIONE DELLA CARICA
(il flusso di corrente su una superficie chiusa è proporzionale alla carica presente)

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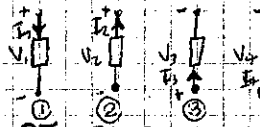
$i = dq/dt$ numero complessivo di portatori di carica



$i = \int_S \vec{J} \cdot \vec{n}\, dS$

Per far muovere la carica ci vuole un campo elettrico (e $\oint \vec{E} \cdot d\vec{s} = 0$ il campo è CONSERVATIVO (campo elettrostatico)

BIPOLO  $V_{AB} = V_A - V_B$



RELAZIONE TENSIONE-CORRENTE
La stabilisce l'interdipendenza tra tensione e corrente

$V = RI$ LEGGE DI OHM ($I = V/R$)

$V = -RI$ → la corrente entra dal morsetto negativo (②-④)

la corrente entra dal morsetto positivo (①-③)

$V_1 = RI_1 \quad V_3 = RI_3 \qquad V_2 = V_1 = RI_1 \quad I_1 = -I_2 \quad V_2 = -RI_2 \qquad V_3 = -RI_1 = R(-I_1) = RI_3 \qquad V_4 = -V_1 = -RI_1 = -RI_4$

$V = f(i)$

CIRCUITO = interconnessione di bipoli



$\oint \vec{J} \cdot \vec{n}\, dS = -dQ/dt$ → flusso uscente se è positivo, flusso entrante se è negativo



$\oint = \int_{S_1} + \int_{S_2} + \int_{S_3} + \int_{S_4} + \int_{S_5} = I_1 \quad I_2 \quad I_3 \quad I_4 \quad 0 \quad y = 0$ (via cava)

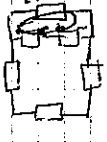
$= I_1 + I_2 + I_3 + I_4 = -dQ/dt$

Flusso attraverso base $\Rightarrow I_1 + I_2 + I_3 + I_4 = 0$

$\Psi \sum_{k=1}^{n} I_k = 0$ PRIMO PRINCIPIO DI KIRCHOFF

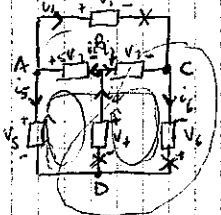
$\oint \vec{E} \cdot d\vec{s} = -d\Phi/dt$ LEGGE DI FARADAY

$\int_1 \vec{E} \cdot d\vec{s} - \int_2 \vec{E} \cdot d\vec{s} + \int_3 \vec{E} \cdot d\vec{s} = 0 \qquad V_1 + V_3 - V_2 = 0$ SECONDO PRINCIPIO DI KIRCHOFF



$V_2 = f(i_2)$





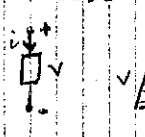
A $i_1 - i_2 + i_5 = 0 \Leftrightarrow i_1 + i_5 = i_2$
B $i_2 + i_3 - i_4 = 0$
C $i_6 - i_1 - i_3 = 0$
D $i_4 - i_5 - i_6 = 0$ → è combinazione delle altre 3, si può cancellare
·) $i_4 - i_5 - i_1 - i_3 = 0$

N NODI ⇒ ho senso scrivere N-1 EQUAZIONI del primo principio di Kirchoff

ACBA $V_1 - V_3 - V_2 = 0$
ABDA $V_2 + V_4 - V_5 = 0$
ABCDA $V_2 + V_3 - V_6 - V_5 = 0$
BCDB $V_3 - V_6 - V_4 = 0$ → è combinazione della seconda e della terza, si può cancellare

L LATI (BIPOLI) 2L INCOGNITE ⇒ { L REL TENSIONE-CORRENTE; N-1 I PK; L-N+1 II PK } / 2L EQ. INDIP.

N NODI



$R = \rho l/S \qquad dR = \rho dl/S(l) \qquad R = \int dR = \int_{l_1}^{l_{finale}} \rho \frac{1}{S(l)} dl$

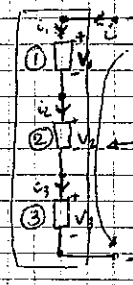
Valgono all'esterno due identici, all'interno no perché la densità di corrente cambia nell'oggetto ⇒ il valore di resistenza descrive quello che accade all'esterno

⇓

EQUIVALENTI AGLI EFFETTI ESTERNI DI STESSA RELAZIONE V-I



COLLEGAMENTO SERIE TRA BIPOLI

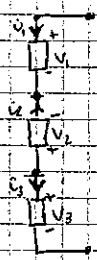


$V_k = R_k i_k \quad k=1,2,3$

$i = i_1 = i_2 = i_3 \qquad V = V_1 + V_2 + V_3 = R_1 i_1 + R_2 i_2 + R_3 i_3 = i\,(R_1 + R_2 + R_3) \rightarrow R_S$

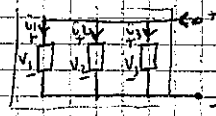
$\rightarrow$ il bipolo serie è una resistenza (rapporto costante tra tensione e corrente)

R_S = bipolo ottenuto dalla serie di tre bipoli



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COLLEGAMENTO PARALLELO TRA BIPOLI



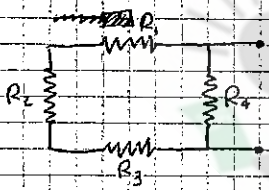
$V_k = R_k i_k \quad i_k = G_k V_k \quad (G_k = 1/R_k)$

$V = V_1 = V_2 = V_3 \qquad i = i_1 + i_2 + i_3 = G_1 V_1 + G_2 V_2 + G_3 V_3 = V\,(G_1 + G_2 + G_3) = G_p V$

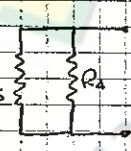
CONDUTTANZA $i = GV \quad G = 1/R \quad G = i/V$

$G_p = \sum G_k$

$1/G_p = R_p = 1/\sum 1/R_k$

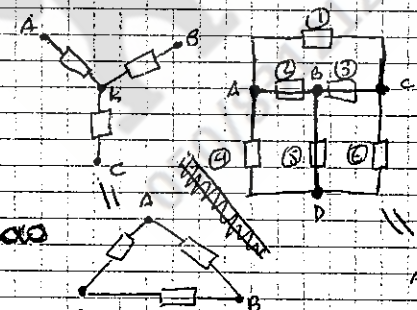


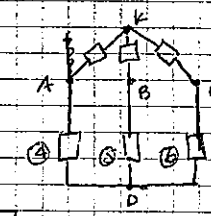
$R_S = R_1 + R_2 + R_3$



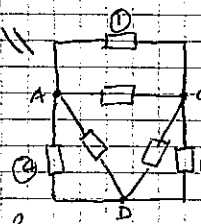
$R_{eq} = \dfrac{1}{\frac{1}{R_S} + \frac{1}{R_4}} = \dfrac{R_S R_4}{R_S + R_4}$

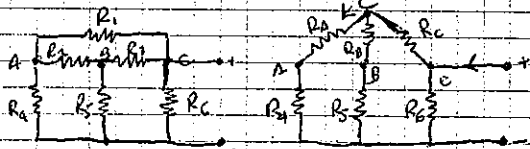
COLLEGAMENTO STELLA



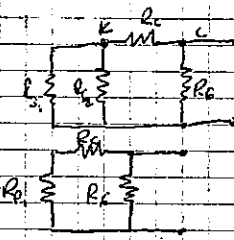


COLLEGAMENTO TRIANGOLO





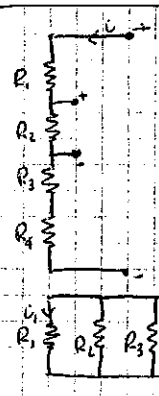
(uguali se sono Δ serie, Δ parallelo)



$R_{S1} = R_A + R_4 \quad R_{S2} = R_B + R_5$

$R_P = \dfrac{R_{S1} R_{S2}}{R_{S1} + R_{S2}}$

$R_{eq} = V/i = \dfrac{R_6 (R_C + R_{P1})}{R_6 + R_C + R_{P1}}$



PARTITORE DI TENSIONE <1

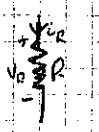
$V_2 = R_2 i_2 = \frac{R_2 V}{R_3}$ $\quad V_2 = \left(\frac{R_2}{R_3}\right) V$

$G_p = \frac{1}{R_p} = \Sigma G_k$

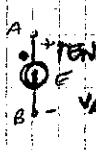
$V = R_p \cdot i = \frac{1}{G_p} i \qquad i_1 = G_1 V_1 = \frac{G_1}{G_p} i \qquad i_1 = \left(\frac{G_1}{\Sigma G_k}\right) i$

PARTITORE DI CORRENTE

GENERATORI IDEALI INDIPENDENTI



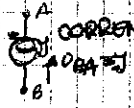
$V_R = R i_R$



TENSIONE

$V_{AB} = E$, IDEALE E INDIPENDENTE

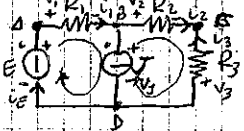
↳ se E è assegnato a priori



CORRENTE → se J vale e funzione di V_{AB}

$i_{BA} = J$ IDEALE E INDIPENDENTE

↳ se J è una funzione del tempo assegnata



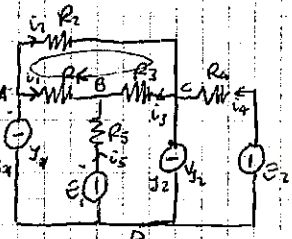
5 BIPOLI → 8 INCOGNITE

$$\begin{cases} V_1 = R_1 i_1 \\ V_2 = R_2 i_2 \\ V_3 = R_3 i_3 \\ A \quad i_E = i_1 \\ B \quad i_1 + J = i_2 \\ C \quad i_2 + i_3 = 0 \\ ABDA \quad V_1 + V_J - E = 0 \\ BCDB \quad V_2 - V_3 - V_J = 0 \end{cases}$$

$i_E = i_E = i_1 \qquad i_J = i_2 = -i_3$

$$\begin{cases} i_x + J = i_J \\ ABCDA \quad R_1 i_x + R_2 i_J + R_3 i_J - E = 0 \\ ABDA \quad R_1 i_x + V_J - E = 0 \end{cases}$$

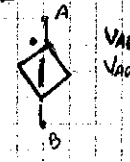
$V_{AB} + V_{BD} + V_{DA} = 0$



$$\begin{cases} A \quad J_1 = i_1 + i_2 \\ B \quad i_1 + i_5 + i_3 = 0 \\ C \quad i_4 + i_2 - i_3 + J_2 = 0 \\ ABDA \quad R_2 i_2 + R_3 i_3 - R_1 i_1 = 0 \\ CBDC \quad R_3 i_3 - R_5 i_5 + E_1 - E_2 + R_4 i_4 = 0 \\ -V_{J_1} + R_1 i_1 - R_5 i_5 + E_1 = 0 \\ -V_{J_2} + R_3 i_3 - R_5 i_5 + E_1 = 0 \end{cases}$$

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GENERATORI IDEALI CONTROLLATI → la tensione o la corrente sono funzioni non del tempo, ma di altri elementi del circuito.



$V_{AB} = \varphi(i_k(t), t)$

$V_{AB} = \alpha i_k(t)$



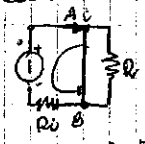
$i_{BA}(t) = \varphi(i_k(t), V_h(t))$

$i_{BA}(t) = \alpha_1 i_k(t) + \alpha_2 V_h(t)$

GENERATORI REALI

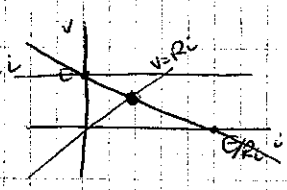
$V_{AB}(t) = e(t) - \varphi(i_{AB}(t))$ → la tensione dipende dalla corrente

$i = \frac{e}{R + R_i} \rightarrow I_{cc} = \frac{E}{R_i}$ $\quad R_i$ RESISTENZA INTERNA IN SERIE



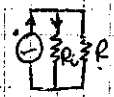
$V_{AB} = E - R_i i$

$R_i \ll R_{est}$

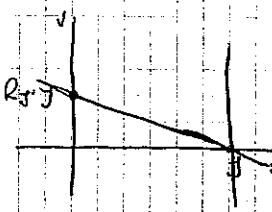


$i = \frac{E}{R_i + R_{est}} \simeq \frac{E}{R_{est}}$

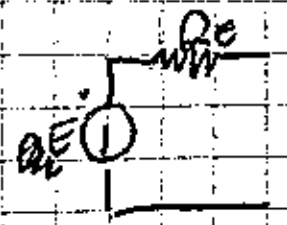
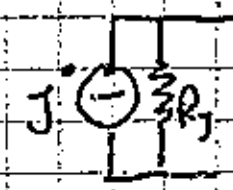
$i_{AB} = J(t) + \varphi(V_{AB})$ $\quad i = J - \frac{V}{R_i}$ $\quad R_i$ RESISTENZA INTERNA IN PARALLELO



$R_i \gg R_{est}$



$V_{AB} = J \frac{R_i R_{est}}{R_i + R_{est}} \simeq J R_{est}$

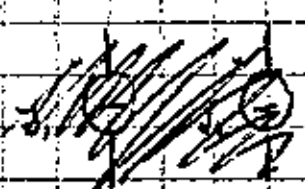



$i = J - \frac{V}{R_J}$

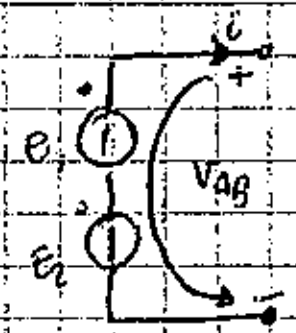
$V = E - R_E i$

$E = R_J J \quad J = E/R_E \Rightarrow$ i due bipoli sono equivalenti agli effetti esterni (stessa relazione tensione corrente)
$R_J = R_E = R \quad J = E/R$

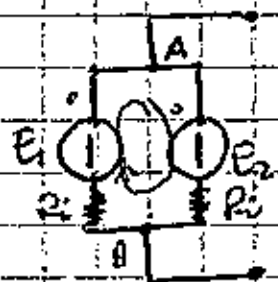
$R = 0 \quad R = \infty \Rightarrow$ l'equivalenza non esiste!!



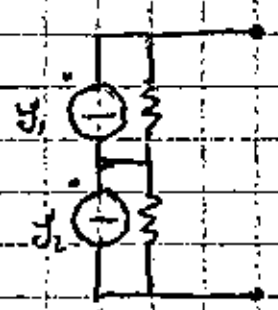
COLLEGAMENTI TRA GENERATORI



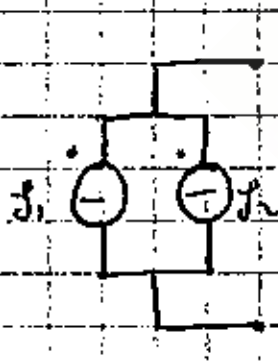
SERIE
$V_{AB} = E_1 + E_2 \Rightarrow$ se i generatori sono ideali, equivale a un generatore ideale



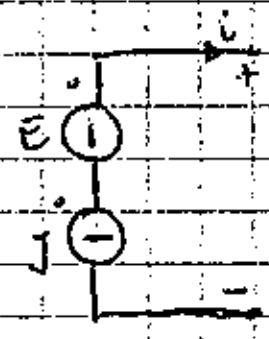
PARALLELO $\rightarrow$ ha senso in modo ideale solo se $E_1 = E_2$
$E_2 - E_1 + R_i I + R_i I = 0 \quad I = \frac{E_1 - E_2}{2R_i}$



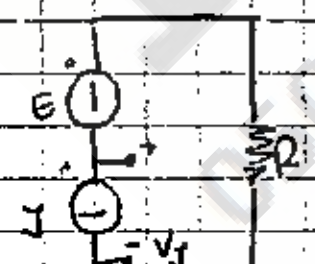
SERIE $\rightarrow$ ha senso in modo ideale solo se $J_1 = J_2$



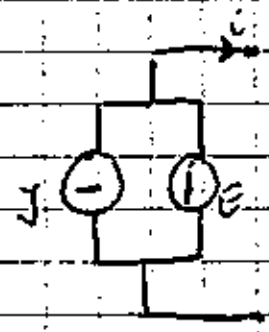
PARALLELO
$i = J_1 + J_2 \rightarrow$ se i generatori sono ideali, equivale a un generatore ideale



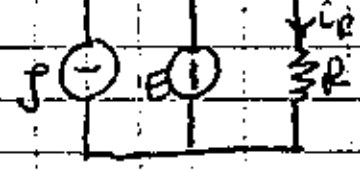
SERIE $\rightarrow$ è equivalente agli effetti esterni a un generatore di corrente
$i = J = i$



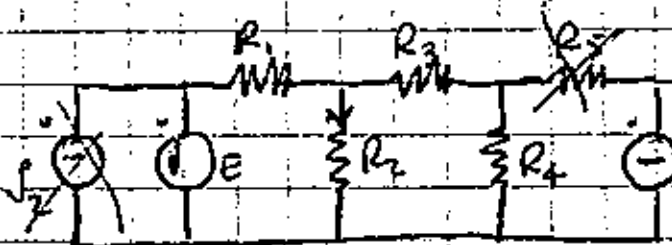
$J = 10 \quad R = 1 \quad E = 5 \quad i_E = 10 \quad V_R = 10 \quad V_J = -E + RJ$ → conta solo alla determinazione di V_J



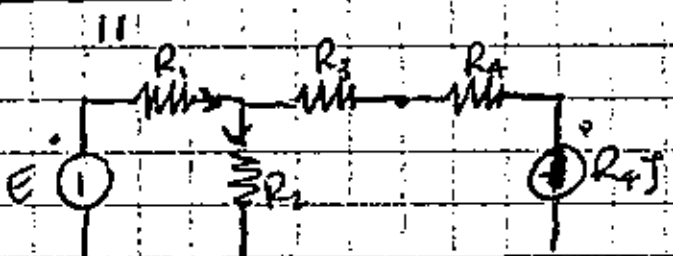
PARALLELO $\rightarrow$ è equivalente agli effetti esterni a un generatore di tensione
$V = E$



$i_R = E/R \quad i_E = i_R - J = E/R - J$ → conta solo alla determinazione di i_E



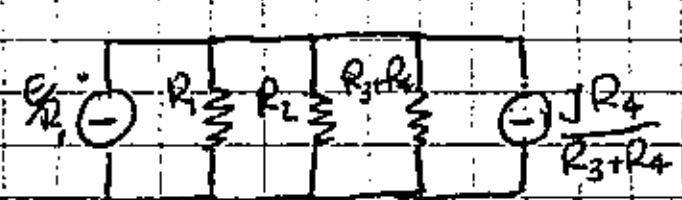
J e R_5 sono in serie $\Rightarrow$ agli effetti esterni equivale a J e basta $\Rightarrow$ si elimina R_5
J_2 e E sono in parallelo $\Rightarrow$ " " " " E e basta



$I_E = I_{R_1} - J$



$I_{R_1} = I_{R_2} + I_{R_{3,4}} - \frac{R_4 J}{R_3 + R_4}$



$I_{R_2} = \left(\frac{E}{R_1} + \frac{J R_4}{R_3 + R_4}\right) \frac{1/R_2}{1/R_1 + 1/R_2 + 1/(R_3+R_4)}$

$I_{R_{3,4}} = \left(\frac{E}{R_1} + \frac{J R_4}{R_3 + R_4}\right) \frac{1/(R_3+R_4)}{1/R_1 + 1/R_2 + 1/(R_3+R_4)}$

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